

**Tentamen Vaste Stof Fysica I**  
29 november 2000

Please write your name, date of birth, enrolment date and student number on each sheet.

**U kunt de vragen in het Nederlands beantwoorden.**

Question 1 (30%)

Consider a system of electrons moving in a two-dimensional plane. The electron density is  $n$ ,  $e$  is their charge,  $m$  is their mass and  $\tau$  is their collision time.

- a) If we now apply an electrical field  $\vec{E}$  parallel to the plane, this will induce a current density  $\vec{j}$ . Derive the expression for  $\sigma_{xx} = j_x / E_x$  from the classical equations of motion.
- b) We now also apply a magnetic field  $\vec{B}$  perpendicular to the plane. The force on an electron in the crossed electric and magnetic fields is:  $\vec{F} = -e(\vec{E} + \vec{v} \times \vec{B})$  where  $\vec{v}$  is the drift velocity of the electrons. The Hall constant is  $R_H = \frac{E_y}{B_z j_x}$ . Derive the expression for the Hall constant in terms of the electron density.
- c) Let us now completely neglect the scattering, with only the magnetic field applied. What kind of motion do the electrons make in this case? Derive the expression for the characteristic frequency,  $\omega_c$ , of this motion. Describe an experiment with which  $\omega_c$  can be measured.
- d) Now consider a 3-dimensional piece of metal, say Al, with electron density  $n$ . Derive the frequency  $\omega_p$  for the free oscillation of the electron plasma, using classical equations of motion. Use the accompanying figure of a slab of material, and consider the field inside the slab, generated by displacing all electrons over a distance  $u$  with respect to the positive background. How is  $\omega_p$  called?

Question 2 (15%)

The non-existing compound  $\text{KnBy}$  has a simple cubic lattice with a basis consisting of a  $\text{Kn}^+$  ion at coordinate  $(0,0,0)$  and a  $\text{By}^-$  ion at  $(1/2, 1/2, 1/2)$ . The lattice parameter is  $a$ . The formfactors of  $\text{Kn}^+$  and  $\text{By}^-$  have the ratio  $f_{\text{Kn}^+}/f_{\text{By}^-} = 3/2$

- 3a. Determine for which reciprocal lattice vectors  $\mathbf{G} = v_1\mathbf{b}_1 + v_2\mathbf{b}_2 + v_3\mathbf{b}_3$  reflections will be observable in the X-ray diffraction spectrum.
- 3b. Calculate the intensity  $I = |S(v_1, v_2, v_3)|^2$  of these diffraction spots. For which values of  $(v_1, v_2, v_3)$  is the intensity high, and for which ones low?
- 3c. What can you say about the X-ray diffraction spectrum  $f_{\text{Kn}^+}/f_{\text{By}^-} = I$ ?

Hint: The structure factor is  $S(\mathbf{G}) = \sum_j f_j \exp(-i\mathbf{G} \cdot \mathbf{r}_j)$ .

$\mathbf{r}_j$  is the position of the  $j$ -th atom in the conventional unit cell.

Question 3 (15%)

The compound  $\text{NaV}_2\text{O}_5$  has an orthorhombic structure, which means that  $|\mathbf{a}_1| \neq |\mathbf{a}_2| \neq |\mathbf{a}_3|$ , and the basisvectors are orthogonal to each other. The unit cell contains two formula units, i.e. 2 Na atoms, 4 V-atoms and 10 oxygen atoms.

- a. How many accoustical and how may optical branches of phonons do you expect?
- b. What is the difference between accoustical and optical phonons?
- c. What is the difference between a transverse and longitudinal phonon?
- d. Do you expect that the three branches of accoustical phonons of  $\text{NaV}_2\text{O}_5$  for sound propagating along the  $[100]$  direction have the same sound velocity? Why?

Question 4 (30%)

a) What is, using Hund's rules, the groundstate of the following free ions:

(i)  $V^{3+}$ , configuration  $3d^2$ .

(ii)  $Ru^{3+}$ , configuration  $4d^5$ .

(iii)  $Pu^{2+}$ , configuration  $5f^6$ .

Use spectroscopic notation. Example: For  $Eu^{2+}(4f^2)$  the answer would have been  $^8S_{7/2}$ .

b) In the solid  $CaV_2O_5$  the  $V^{4+}$ -ion has a  $3d^1$  configuration. The magnetic moment of  $V^{4+}$  is not the Hund's rule value  $p = g\sqrt{J(J+1)}$  but it is  $p = 2\sqrt{S(S+1)}$  with  $S = 1/2$ .

Explain why there is no orbital contribution to the paramagnetism of  $V^{4+}$  in this compound. Why is the full spin+orbital paramagnetism corresponding to the  $^3H_4$  Hund's rule state of  $U^{4+}(5f^2)$  observed in the magnetic susceptibility of  $UO_2$ ?

c) The magnetic susceptibility of a paramagnetic substance is given by the Curie law

$$\chi_0 = \frac{M}{B} = \frac{Np^2\mu_B^2}{3k_B T} = \frac{C}{T}.$$

Consider a ferromagnet assuming the effective medium approximation. As a result the surrounding paramagnetic atoms cause an exchange field  $B_E = \lambda M$  acting on every moment, in addition to the applied field  $B_a$ . Use Curie's law to derive an expression for the effective susceptibility in mean field approximation

$$\chi = \frac{M}{B_a}.$$

d) Consider the case of Nickel, where  $\lambda \approx 3300$ , and the Curie constant  $C = 0,19K$ . At which temperature Ni orders Ferromagnetically?

e) What are magnons? Sketch the dispersion relation.